

PRACTICE AND APPLICATIONS

STUDENT HELP

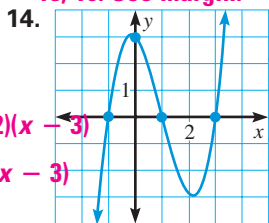
► **Extra Practice**
to help you master
skills is on p. 949.

15. $f(x) = -\frac{1}{2}(x + 1)(x - 2)(x - 3)$

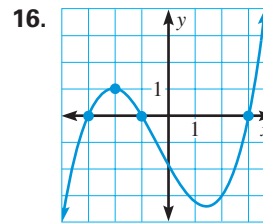
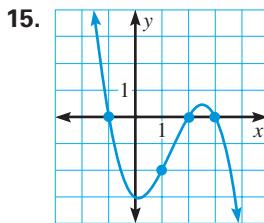
16. $f(x) = \frac{1}{5}(x + 3)(x + 1)(x - 3)$

WRITING CUBIC FUNCTIONS Write the cubic function whose graph is shown.

15, 16. See margin.



$f(x) = (x + 1)(x - 1)(x - 3)$



FINDING A CUBIC MODEL Write a cubic function whose graph passes through the given points. 17–22. See margin.

17. $(-1, 0), (-2, 0), (0, 0), (1, -3)$

18. $(3, 0), (2, 0), (-3, 0), (1, -1)$

19. $(1, 0), (3, 0), (-2, 0), (2, 1)$

20. $(-1, 0), (-4, 0), (4, 0), (0, 3)$

21. $(3, 0), (2, 0), (-1, 0), (1, 4)$

22. $(0, 0), (-3, 0), (5, 0), (-2, 3)$

FINDING FINITE DIFFERENCES Show that the n th-order differences for the given function of degree n are nonzero and constant.

23. $f(x) = x^2 - 3x + 7$ 24. $f(x) = 2x^3 - 5x^2 - x$ 25. $f(x) = -x^3 + 3x^2 - 2x - 3$
26. $f(x) = x^4 - 3x^3$ 27. $f(x) = 2x^4 - 20x$ 28. $f(x) = -4x^2 + x + 6$
29. $f(x) = -x^4 + 5x^2$ 30. $f(x) = 3x^3 - 5x^2 - 2$ 31. $f(x) = -3x^2 + 4x + 2$



FINDING A MODEL Use finite differences and a system of equations to find a polynomial function that fits the data. You may want to use a calculator.

32.

x	1	2	3	4	5	6
$f(x)$	-4	0	10	26	48	76

$$f(x) = 3x^2 - 5x - 2$$

34.

x	1	2	3	4	5	6
$f(x)$	-4	-6	-2	14	48	106

$$f(x) = x^3 - 3x^2 - 2$$

36.

x	1	2	3	4	5	6
$f(x)$	-3	-8	-15	-21	-23	-18

$$f(x) = 0.5x^3 - 4x^2 + 3.5x - 3$$

38.

x	1	2	3	4	5	6
$f(x)$	-5	0	9	16	15	0

$$f(x) = -x^3 + 8x^2 - 12x$$

40.

x	1	2	3	4	5	6
$f(x)$	20	-2	-4	2	4	-10

$$f(x) = -2x^3 + 22x^2 - 74x + 74$$

42.

x	1	2	3	4	5	6
$f(x)$	26	-4	-2	2	2	16

$$f(x) = x^4 - 15x^3 + 81x^2 - 183x + 142$$

33.

x	1	2	3	4	5	6
$f(x)$	17	28	33	32	25	12

$$f(x) = -3x^2 + 20x$$

35.

x	1	2	3	4	5	6
$f(x)$	-2	-6	-6	4	30	78

$$f(x) = x^3 - 4x^2 + x$$

37.

x	1	2	3	4	5	6
$f(x)$	2	20	58	122	218	352

$$f(x) = x^3 + 4x^2 - x - 2$$

39.

x	1	2	3	4	5	6
$f(x)$	-2	1	-4	-5	10	53

$$f(x) = 2x^3 - 16x^2 + 37x - 25$$

41.

x	1	2	3	4	5	6
$f(x)$	2	-5	-4	-1	-2	-13

$$f(x) = -x^3 + 10x^2 - 30x + 23$$

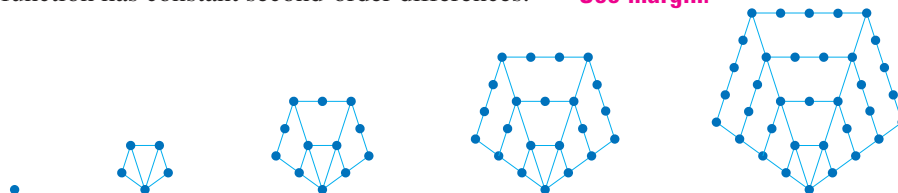
43.

x	1	2	3	4	5	6
$f(x)$	0	6	2	6	12	-10

$$f(x) = -x^4 + 13x^3 - 58x^2 + 104x - 58$$

44. **PENTAGONAL NUMBERS** The dot patterns show pentagonal numbers. A

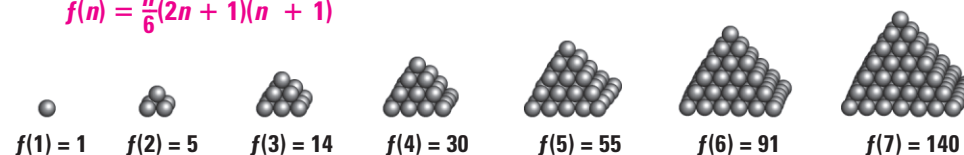
formula for the n th pentagonal number is $f(n) = \frac{1}{2}n(3n - 1)$. Show that this function has constant second-order differences. **See margin.**



45. **HEXAGONAL NUMBERS** A formula for the n th hexagonal number is $f(n) = n(2n - 1)$. Show that this function has constant second-order differences. **See margin.**

46. **SQUARE PYRAMIDAL NUMBERS** The first six square pyramidal numbers are shown. Find a polynomial function that gives the n th square pyramidal number.

$$f(n) = \frac{n}{6}(2n + 1)(n + 1)$$





FINDING MODELS In Exercises 47–49, use a graphing calculator to find a polynomial model for the data.

47. **GIRL SCOUTS** The table shows the number of Girl Scouts (in thousands) from 1989 to 1996. Find a polynomial model for the data. Then estimate the number of Girl Scouts in 2000. **See margin.**

t	1989	1990	1991	1992	1993	1994	1995	1996
y	231	253.3	273.8	284.1	294.1	303.6	368.6	383.7

See margin.

48. **REAL ESTATE** The table shows the average price (in thousands of dollars) of a house in the Northeastern United States for 1987 to 1995. Find a polynomial model for the data. Then predict the average price of a house in the Northeast in 2000. **DATA UPDATE** of *Statistical Abstract of the United States* data at www.mcdougallittell.com

x	1987	1988	1989	1990	1991	1992	1993	1994	1995
$f(x)$	140	149	159.6	159	155.9	169	162.9	169	180

49. **SPACE EXPLORATION** The table shows the average speed y (in feet per second) of a space shuttle for different times t (in seconds) after launch. Find a polynomial model for the data. When the space shuttle reaches a speed of approximately 4400 feet per second, its booster rockets fall off. Use the model to determine how long after launch this happens. **See margin.**

t	10	20	30	40	50	60	70	80
y	202.4	463.4	748.2	979.3	1186.3	1421.3	1795.4	2283.5

50. **MULTI-STEP PROBLEM** Your friend has a dog-walking service and your cousin has a lawn-care service. You want to start a small business of your own. You are trying to decide which of the two services you should choose. The profits for the first 6 months of the year are shown in the table. **50.a, b. See margin.**

Dog-walking service	Month, t	1	2	3	4	5	6
	Profit, P	3	5	22	54	101	163
Lawn-care service	Month, t	1	2	3	4	5	6
	Profit, P	3	21	41	68	107	163

- a. Use finite differences to find a polynomial model for each business.
 b. **Writing** You want to choose the business that will make the greater profit in December (when $t = 12$). Explain which business you should choose and why.

51. a. Substitute the expressions $x, x + 1, x + 2, \dots, x + 5$ for x in the function $f(x) = ax^3 + bx^2 + cx + d$ and show that third-order differences are constant. **51.a, b. See margin.**

- b. The data below can be modeled by a cubic function. Set the variable expressions you found in part (a) equal to the first-, second-, and third-order differences for these values. Solve the equations to find the coefficients of the function that models the data. Check your work by substituting the original data values into the function.

x	1	2	3	4	5	6
$f(x)$	-1	1	-3	-7	-5	9



EILEEN COLLINS

was selected by NASA for the astronaut program in 1990. Since then she has become the first woman to pilot a spacecraft and the first woman to command a space shuttle.

47. $f(t) = 0.641t^3 - 4.93t^2 + 25.8t + 232$ where t is the number of years since 1989; 772,000 Girl Scouts

Test Preparation

48. $y = 0.242t^3 - 3.00t^2 + 13.5t + 140$ where t is the number of years since 1987; about \$340,000

49. $y = 0.007t^3 - 0.740t^2 + 49.0t - 236$; about 101 sec

Challenge

EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

58. $-9, -3$

59. $-3 \pm \sqrt{33}$

60. $-3, 6$

61. $-2 \pm \frac{i\sqrt{6}}{2}$

62. $-1, 15$

63. $3 \pm \frac{i\sqrt{15}}{3}$

64. $(2x - 1)(4x^2 + 2x + 1)$

65. $(3x + 2)(9x^2 - 6x + 4)$

66. $8(3x + 2)(9x^2 - 6x + 4)$

67. $(2x - 5)(4x^2 + 10x + 25)$

68. $3(x - 2)(x^2 + 2x + 4)$

SOLVING QUADRATIC EQUATIONS Solve the equation. (Review 5.3 for 7.1)

52. $3x^2 = 6 \pm \sqrt{2}$

53. $16x^2 = 4 \pm \frac{1}{2}$

54. $4x^2 - 5 = 9 \pm \frac{\sqrt{14}}{2}$

55. $6x^2 + 3 = 16 \pm \frac{\sqrt{78}}{6}$

56. $-x^2 + 9 = 2x^2 - 6 \pm \sqrt{5}$

57. $-x^2 + 2 = x^2 + 1 \pm \frac{\sqrt{2}}{2}$

SOLVING EQUATIONS Solve the equation by completing the square. (Review 5.5)

58. $x^2 + 12x + 27 = 0$

59. $x^2 + 6x - 24 = 0$

60. $x^2 - 3x - 18 = 0$

61. $2x^2 + 8x + 11 = 0$

62. $-x^2 + 14x + 15 = 0$

63. $3x^2 - 18x + 32 = 0$

SUM OR DIFFERENCE OF CUBES Factor the polynomial. (Review 6.4)

64. $8x^3 - 1$

65. $27x^3 + 8$

66. $216x^3 + 64$

67. $8x^3 - 125$

68. $3x^3 - 24$

69. $8x^3 + 216$

70. $27x^3 + 1000$

71. $3x^3 + 81$